

Coil Length-to-Diameter Ratios For Maximum Q in LFMF Antenna Loading Inductors

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Introduction

Typical amateur LFMF antennas need a large loading inductor and in most cases inductor loss will be a major portion of the total antenna loss. To achieve reasonable radiation efficiency high Q inductors are needed. One way to obtain higher Q is to make the inductor physically larger increasing diameter, length, conductor size, etc, but there's problem. If you start with a small coil with a given inductance and geometry and scale up its dimensions, keeping the inductance constant, initially Q will increase nicely but simultaneously the self-resonant frequency (SRF) will decrease. At some point SRF will limit the achievable Q. Length-to-diameter ratio (L/D) and SRF are interrelated. L/D also affects proximity loss complicating the search.

The need for maximum inductor Q is as old as radio itself and represents a very long running discussion. One consistent element of that discussion is the question of optimum winding L/D. If the operating frequency is low compared to SRF ($f/\text{SRF} < 0.2-0.3$) then the answer is simple, optimum $L/D=0.45$ or close to it, if not, the answer is much more complicated. Unfortunately because large inductance values often needed and physically large inductors are used to obtain high Q the effect of SRF usually cannot be ignored.

We have reliable equations for calculating both skin and proximity effects and SRF. However, some of these expressions are algebraically messy and interrelated in complicated ways. As a practical matter finding optimum Q solutions, taking into account all the variables, can be a bear. Fortunately Brian Beezley, K6STI, has created an inductor design program^[1] (COIL) which includes all the variables and equations combined with an optimizer. This is a great boon and I have used Brian's program for this discussion.

Loading inductors are usually a single layer circular air core solenoid wound with round wire. Of course there are many other construction options but the single layer coil is typical and the focus of this work.

Definitions

Figure 1 defines some of the variables.

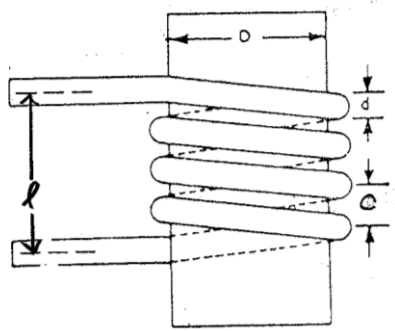


Figure 1 - inductor dimensions

$c \rightarrow$ center-to-center spacing between turns

$d \rightarrow$ diameter of the winding conductor

$D \rightarrow$ diameter of the winding

$l \rightarrow$ coil length

$L \rightarrow$ actual inductance at the operating frequency

$L_o \rightarrow$ low frequency inductance

$l_w \rightarrow$ length of the winding conductor

$N \rightarrow$ number of turns in the winding

Geometric ratios:

- $(d/c) \rightarrow$ conductor diameter/turn center-to-center spacing ratio
- $(l/D) \rightarrow$ coil length/diameter ratio

An ideal inductor, $f \ll \text{SRF}$

For a very long time RF lore has said that the optimum $l/D \approx 0.5$. Where does that come from, is it valid and if so when? Terman's "Radio Engineers' Handbook"^[2] has been a prime source of RF gospel for over 75 years. On page 74 there is the following statement:

"For given inductance and coil diameter, the Q is a maximum when the ratio (length)(diameter) is of the order of 1/2 /1, although the relation is not at all critical"

No doubt this relationship originated much earlier than Terman but his book is likely the source today. What's the basis for that statement?

If we assume an ideal inductor, used at a frequency where SRF has little impact, a very useful equation for the inductance of a single layer circular winding is:

$$L_o = \frac{DN^2}{18+40(I/D)} \quad [\mu H] \quad (1)$$

This equation comes from Harold Wheeler's 1928 IRE paper^[3] where I and D are in inches and Lo is in uH! For dimensions in cm divide by 2.54. This equation should be correct within 1% for I/D>0.4 and experience shows it's still pretty good down to I/D≈0.2.

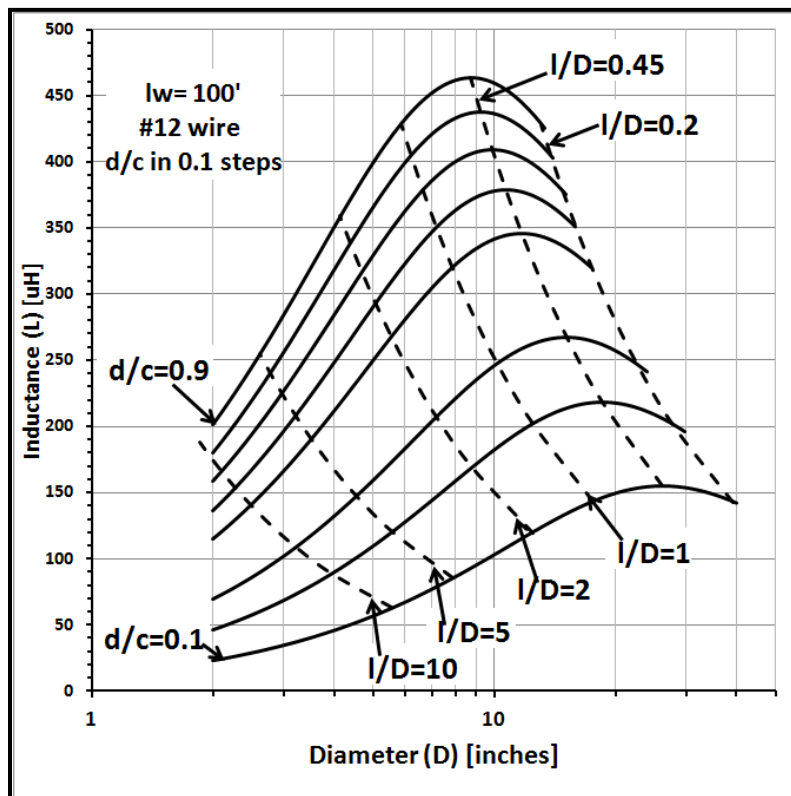


Figure 2 -Inductance for a 100' length of #12 wire.

To better visualize the interrelationships between D, I/D and d/c (turn spacing ratio), we can manipulate Wheeler's equation to be in terms of I/D, D, d, lw and d/c and graph that for a fixed length of wire (lw=100') and wire size (d=0.08"→#12 wire) as shown in figure 2. There are three variables: d/c, D and I/D. The solid lines correspond to values of constant d/c in 0.1 steps. The dashed lines correspond to constant I/D as marked. For all values of d/c the maximum value for Lo occurs at I/D=0.45. I.e. for a given length of wire I/D=0.45 gives you the

maximum inductance. Winding loss (RL) is proportional to the length of the wire in the winding (lw). For skin effect that's true but for proximity effect there is still some effect of I/D. In this case lw is fixed and, assuming proximity loss change with I/D to be small, then I/D≈0.45 should correspond to maximum Q, at least that's the argument. When $f \ll \text{SRF}$ this appears to explain Terman's comment. However, $f \ll \text{SRF}$ is rarely the case for LFMF antenna tuning inductors so we're going to have to look much closer at the effect of SRF and also proximity loss (Kp) both of which are functions of I/D.

Proximity effect

Winding loss due to RL can be represented as the product of the DC resistance (**Rdc**), a factor attributed to skin effect (**Ks**) and a factor associated with proximity effect (**Kp**).

$$RL = R_{dc} \cdot K_s \cdot K_p \quad (2)$$

Ks is not a direct function of I/D but Kp is as illustrated in figure 3. Kp is complicated, changing with N, d/c and increasing for low values of I/D, peaking around I/D≈0.8-1.0 and then decreasing at higher values.

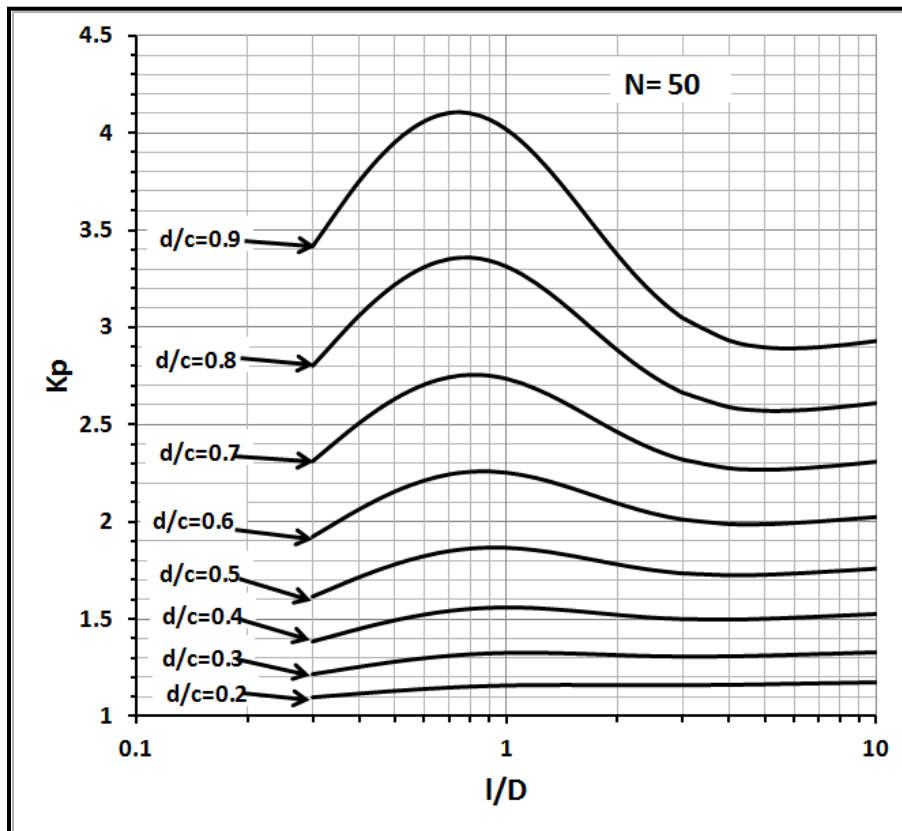


Figure 3 - Kp versus I/D with N=50.

Self resonance

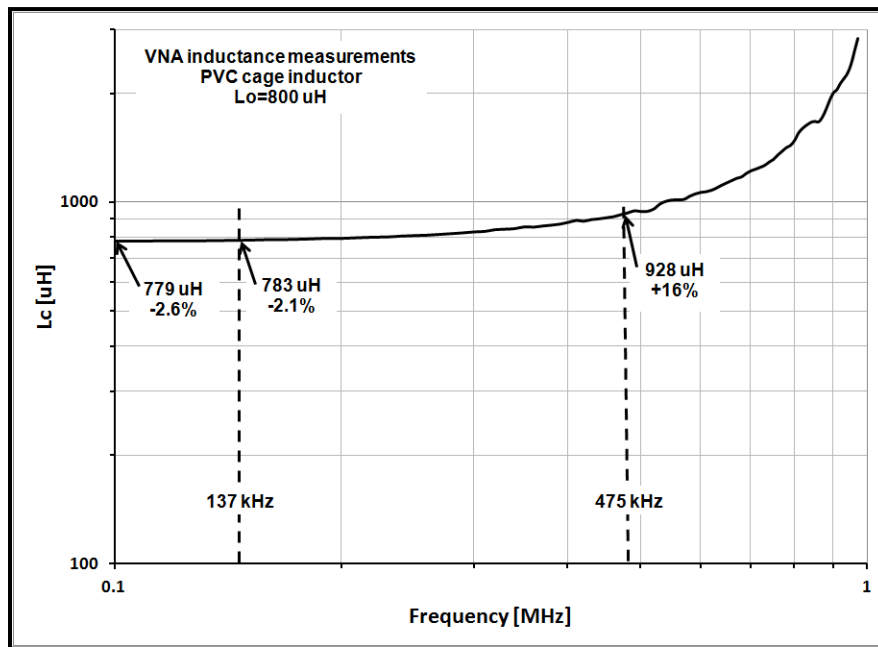


Figure 4- Measured inductance versus frequency example.

Figure 4 is a typical example of measured inductance ($L_0 \approx 800 \mu\text{H}$) over a range of frequencies. At lower frequencies the inductance value is fairly constant and agrees well with Wheeler. However, as we go higher in frequency, L starts to increase, slowly at first but then more rapidly. Since the impedance is rising to a peak this appears to be a parallel resonance which is attributed to self-resonance in the coil.

Besides inductance a coil will have substantial capacitance between the turns and to the immediate surroundings as indicated in figure 5A.

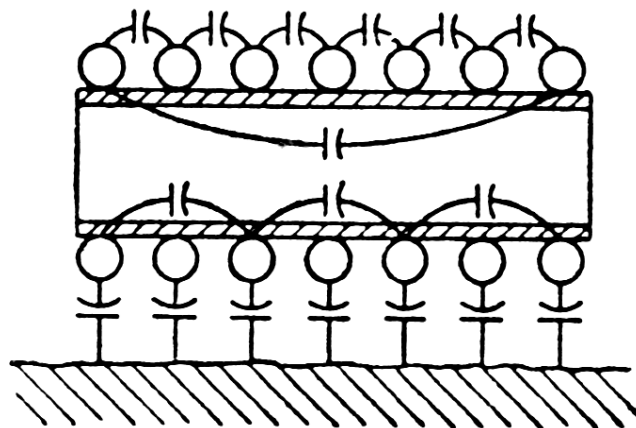


Figure 5A - Distributed capacitance model.

The simplest model (Fig. 5B) lumps the capacitance into an equivalent parallel capacitor.

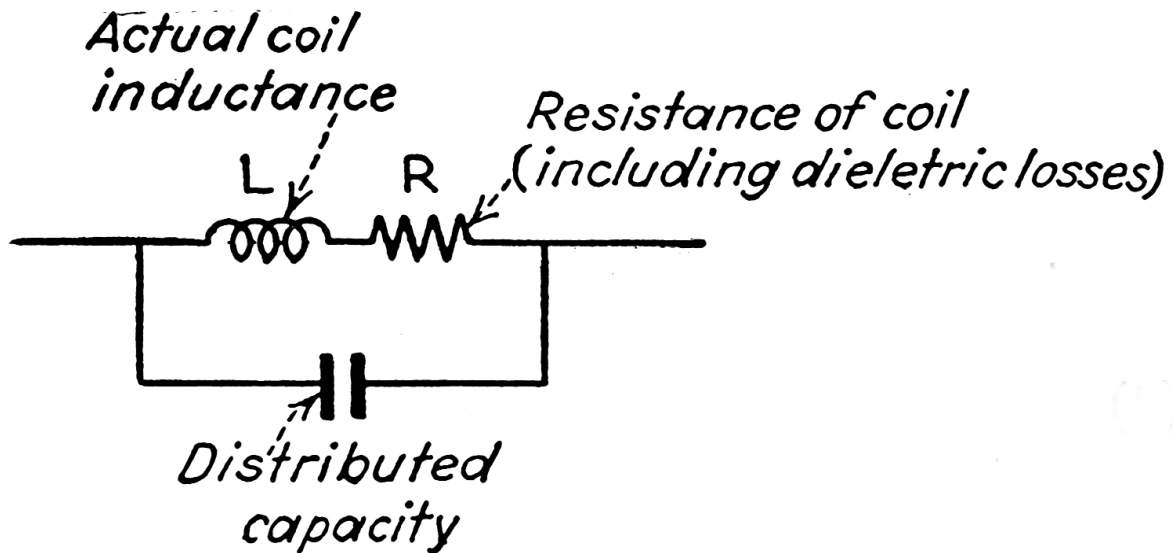


Figure 5B - Inductor equivalent circuit.

Figure 5 presents the traditional explanation of self resonance which works well for many purposes but despite many efforts over the years predicting the distributed capacitance (C_s) using the coil physical dimensions, and from that SRF, has proven to be complicated and not particularly accurate^[7].

If the impedance measurement is extended to frequencies well above the first parallel resonance an alternating series of roughly harmonically related series and parallel resonances are seen. Inductors behave very much like sections of transmission lines. This feature of Inductor self-resonance is nothing new, it was recognized and described as early as 1902^[4] but only recently has a simple equation capable of accurately predicting SRF been derived, primarily from the work of G3RBJ^[5], G3YNH^[6] and KK4VB. Viewing a coil as a transmission line, as shown in figure 6, leads to a simple expression for SRF.

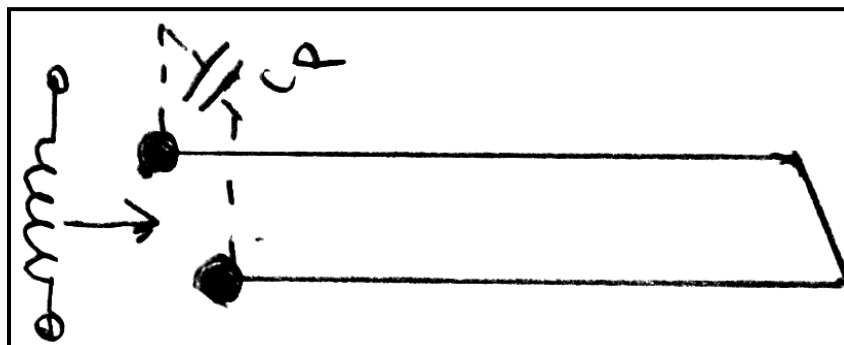


Figure 6- Inductor-transmission line analogy

Note, figure 6 includes the possibility of external shunt parasitic capacitance (C_p) between the coil and it's immediate environment. Equation 3 (given below) does not include C_p but the effect could be included. C_p lowers SRF further!

The basic SRF equation comes from G3RBJ which I have simplified with an approximation that works for $0.3 \leq (l/D) \leq 10$.

For l_w in feet:

$$\text{SRF} = \left(\frac{246}{l_w} \right) [0.37 \ln(l/D) + 1.06] \text{ [MHz]} \quad (3A)$$

For l_w in meters:

$$\text{SRF} = \left(\frac{75}{l_w} \right) [0.37 \ln(l/D) + 1.06] \text{ [MHz]} \quad (3B)$$

Note, these equations are the product to two terms: $\left(\frac{246}{l_w} \right)$ and $\left(\frac{75}{l_w} \right)$ are simply the resonant frequency of a short-circuited $\lambda/4$ length of transmission line and only a function l_w . The second term, graphed in figure 7, is a correction factor that takes into account the geometry of the coil and is only a function of l/D .

$$\text{SRF} = \left(\frac{\lambda}{4} \right) F1 \text{ [MHz]} \quad (4A)$$

Where:

$$F1 = [0.37 \ln(l/D) + 1.06] \text{ [MHz]} \quad (4B)$$

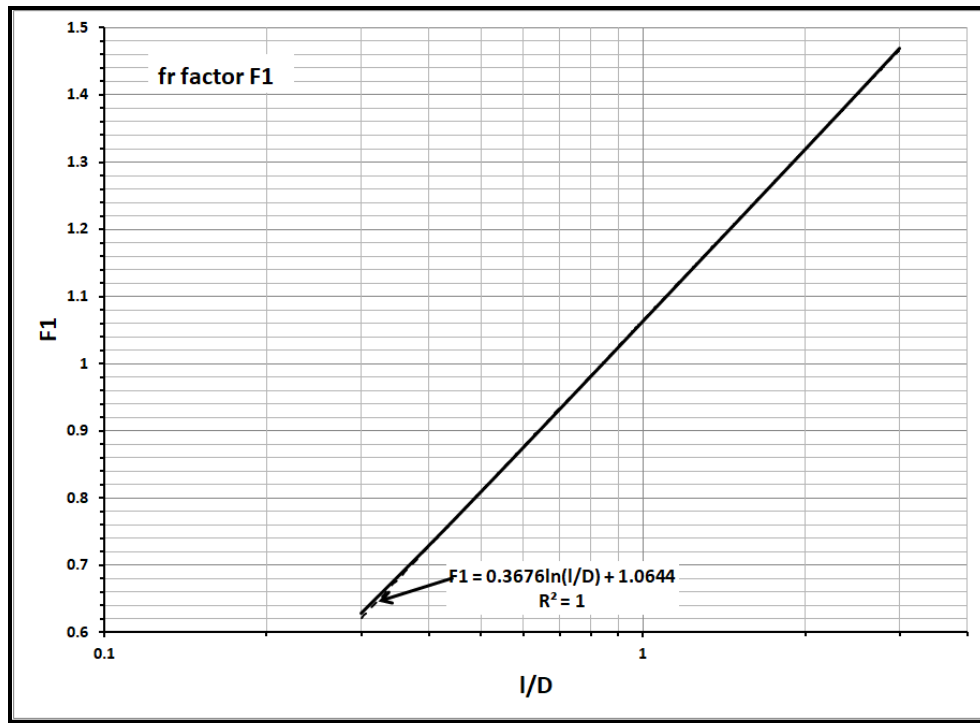


Figure 7 - Factor F1 versus (I/D)

For this discussion the important point to notice that F1 increases with I/D which means SRF increases with I/D. This interaction between SRF and I/D is key to determining an optimum I/D when SRF matters.

Now it's time to look closer at the effect of self-resonance to see why it matters. The change in measured inductance shown in figure 3, as a function of frequency relative to SRF (f/SRF), is shown in figure 8. The equation for this graph is:

$$\frac{L}{L_o} = \frac{1}{1 - \left(\frac{f}{SRF}\right)^2} \quad (5)$$

For $f/SRF=0.45$, $L/L_o=1.25$, there is a 25% increase in inductance above L_o . It should also be noticed that as we increase f/SRF the value of the inductance is changing increasingly rapidly making it harder to predict accurately. L also becomes very sensitive to stray capacitance because of the downward shift in SRF. For $f/SRF < 0.25$ the inductance will be within 5% of L_o so SRF doesn't usually have to be taken into account.

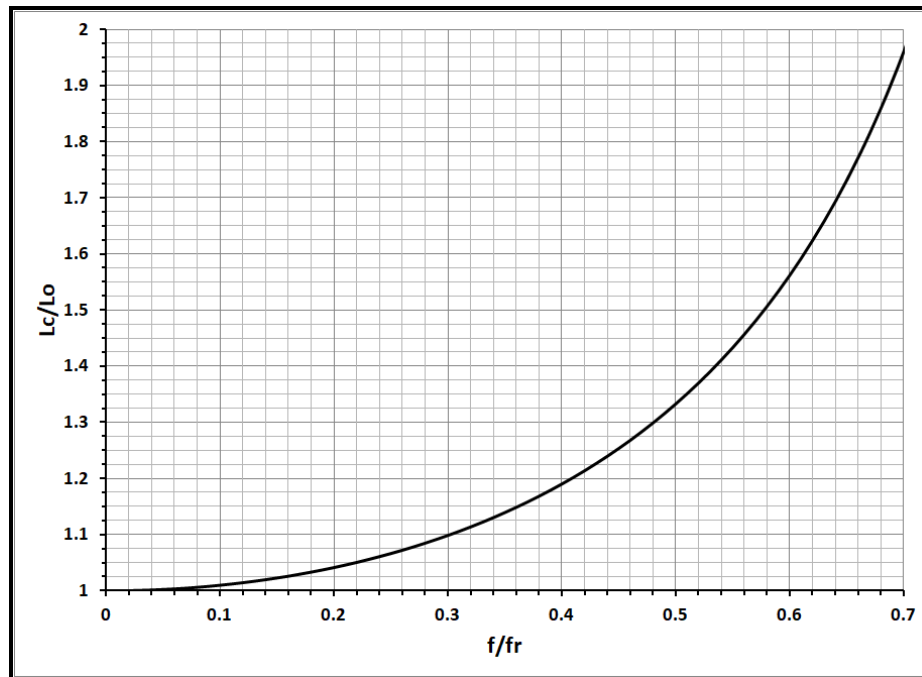


Figure 8 - L_o/L ratio as a function of f/SRF .

Self-resonance also affects Q as shown by measurements of three inductors shown in figure 9. Self-resonance is potentially a Q -killer!

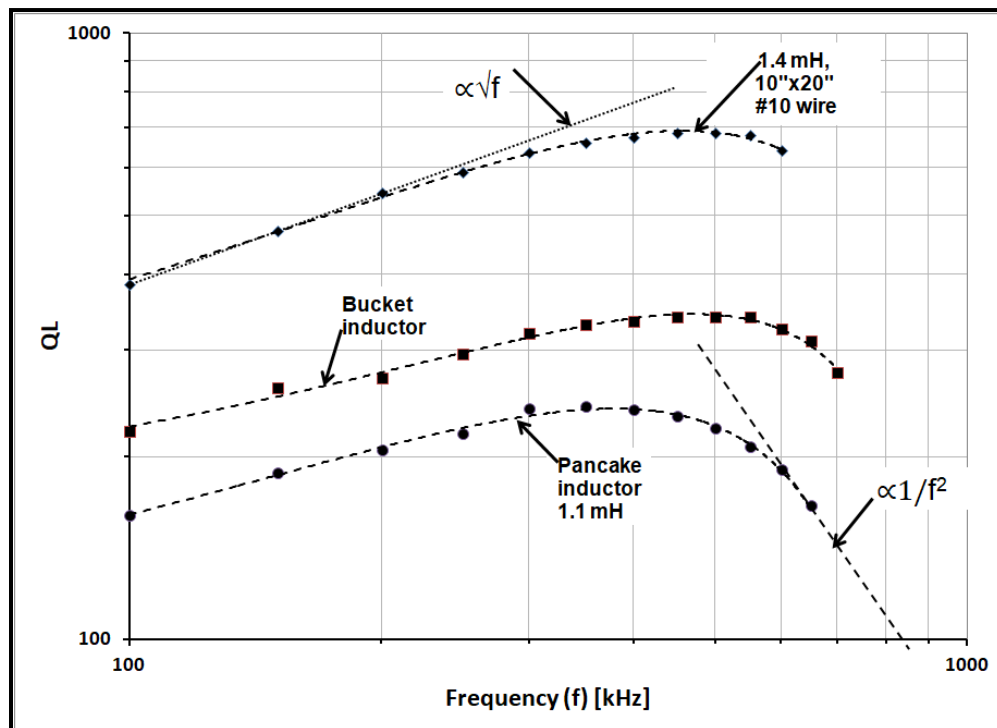


Figure 9-Example of Q variation with frequency.

The Q of an inductor is defined by:

$$Q \equiv \frac{2\pi fL}{R_L} \quad (6)$$

Where Q, L and R_L represent values taking into account K_s, K_p and SRF.

To illustrate the effect of SRF equation 6 can be manipulated into a form which allows us to predict the variation of Q in terms of SRF and a known low frequency Q=Q₁:

$$Q = \left[Q_1 \sqrt{\frac{f}{f_1}} \right] \left[1 - \left(\frac{f}{SRF} \right)^2 \right] \quad (7A)$$

Again, the term "low frequency" with regard to Q₁ is a frequency low enough that SRF does not effect the value. Equation 6A can be rearranged to create a graph of Q versus f relative to fr:

$$Q = \left\{ Q_1 \sqrt{\frac{f_r}{f_1}} \right\} \left\{ \sqrt{\frac{f}{SRF}} \left[1 - \left(\frac{f}{SRF} \right)^2 \right] \right\} = \left\{ Q_1 \sqrt{\frac{SRF}{f_1}} \right\} F \quad (7B)$$

We can graph F to generate a universal curve (shown in figure 10) to illustrate the relationship between Q and SRF in a useful way.

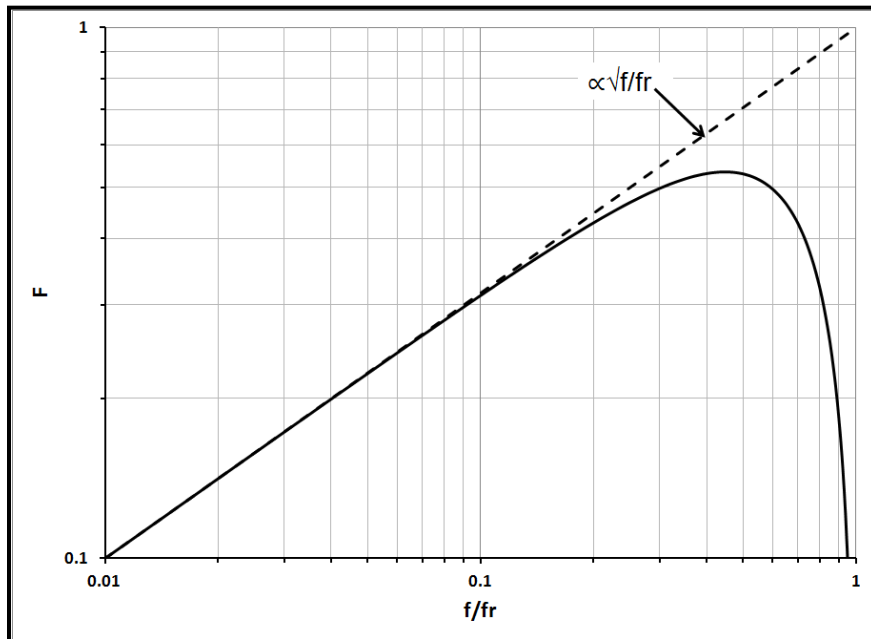


Figure 10 - F-factor versus f/SRF

The dashed line represents how Q changes with frequency when resonance is not present. The solid line represents the change in Q as $f \rightarrow \text{SRF}$. The peak of this graph occurs at $f/\text{SRF}=0.45$. From a large number of coil Q measurements this relationship has been found to be true. Direct measurements of SRF are a tricky business at best but SRF can be estimated from Q measurements, $\text{SRF}=f_{Q\text{-peak}}/0.45$.

SRF, Q and I/D

An example can be used to illustrate the interactions between SRF, Q and I/D . Assume $L_0 = 1$ mH, the winding uses #12 copper wire and a turns $d/c = 0.5$ (one wire diameter space between each turn). As shown in figure 11 we can calculate Q for several different values of I/D . For $f = 137$ kHz SRF has little effect and $I/D = 0.5$ gives the highest Q . But at 475 kHz (keeping $I/D = 0.5$) we are past the peak ($f/\text{SRF} = 0.45$). Stretching out the coil to $I/D = 1$ or 2, SRF increases and the coil Q s are almost as good. However, the story is not that simple. Figure 12 shows the values for the actual inductance as a function of frequency and I/D . Increasing I/D gives more predictable values for L that are less sensitive to C_s .

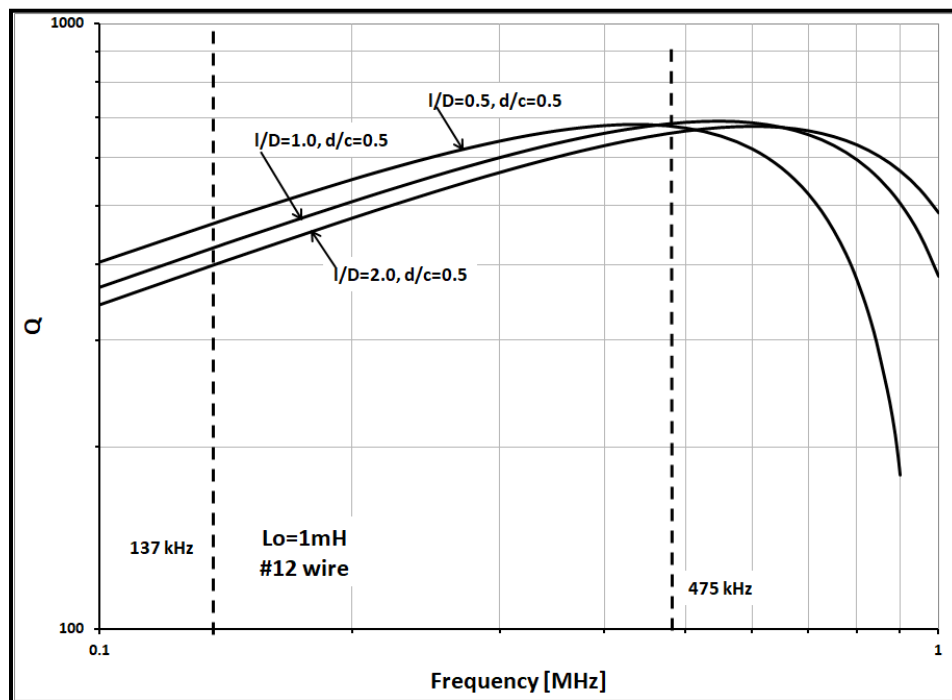


Figure 11 - Q versus frequency with I/D as a parameter.

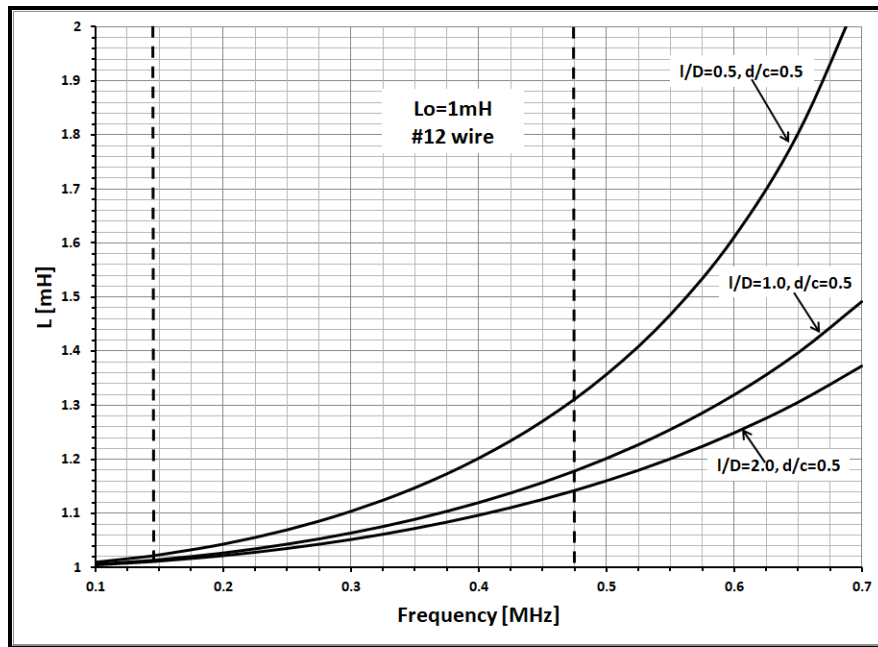


Figure 12 - Inductance versus frequency.

Optimum Q inductors

As the forgoing discussion shows finding the optimum combination of D , I/D and d/c for a given inductance at a given frequency is a complicated business. I have entered all the equations into spread sheets but that's a slow cumbersome process to optimize Q . Using COIL optimum I/D values for typical loading coils can be found quickly. Amateurs are concerned with only very narrow bands: $\approx 137 \text{ kHz}$ and $\approx 475 \text{ kHz}$. Only few wire sizes are generally used: #14, #12 and #10. In addition the ranges for inductor values likely to be needed are limited, typically:

At 475 kHz, L varies from $\approx 400 \mu\text{H} \rightarrow 1200 \mu\text{H}$.

At 137 kHz, L varies from $\approx 1 \text{ mH} \rightarrow 10 \text{ mH}$.

Focusing on amateur LFMF applications greatly narrows the problem!

Inductor examples

As an experiment let's design a series of inductors with Q optimized and note the associated I/D . At 475 kHz, let $L = 400\text{--}1200 \text{ uH}$ and at 137 kHz, $L = 1\text{--}10 \text{ mH}$. Three wire sizes will be used: #14, #12 and #10. Each trial inductor will be optimized for a given frequency and wire size with no restrictions on D , I or d/c . The goal is maximum Q . Using Coil and optimizing Q we get the results shown in figure 13.

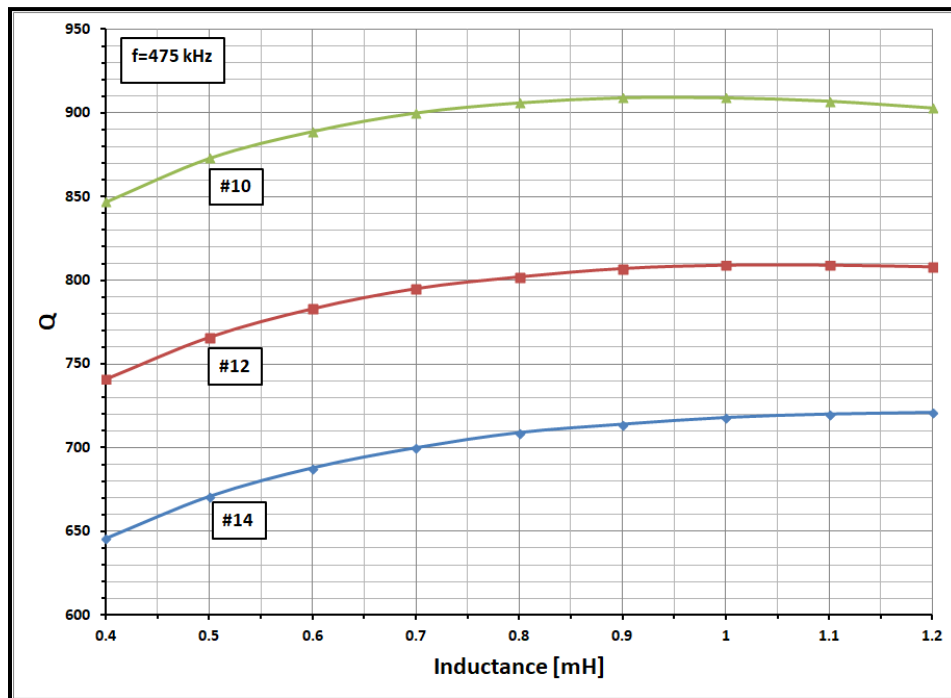


Figure 13A - Optimum Q at 475 kHz.

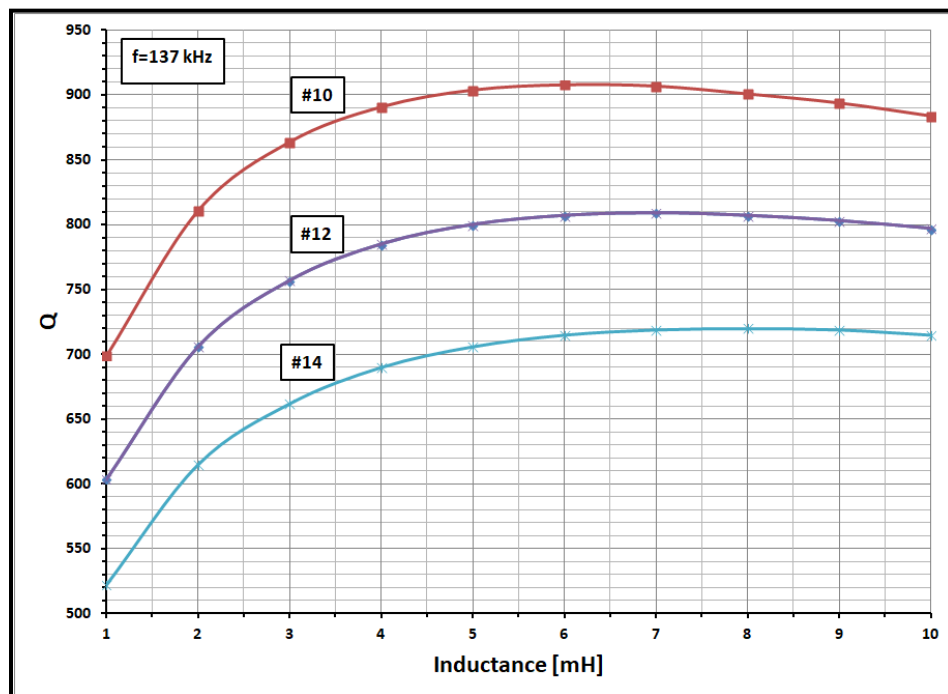


Figure 13B - Optimum Q at 137 kHz.

I/D associated with optimum Q

Figure 14 shows the I/D ratios associated with the 137 and 475 kHz coils. Clearly I/D optimum is not 0.45 except for smaller inductances! As L increases I/D rises towards 1.

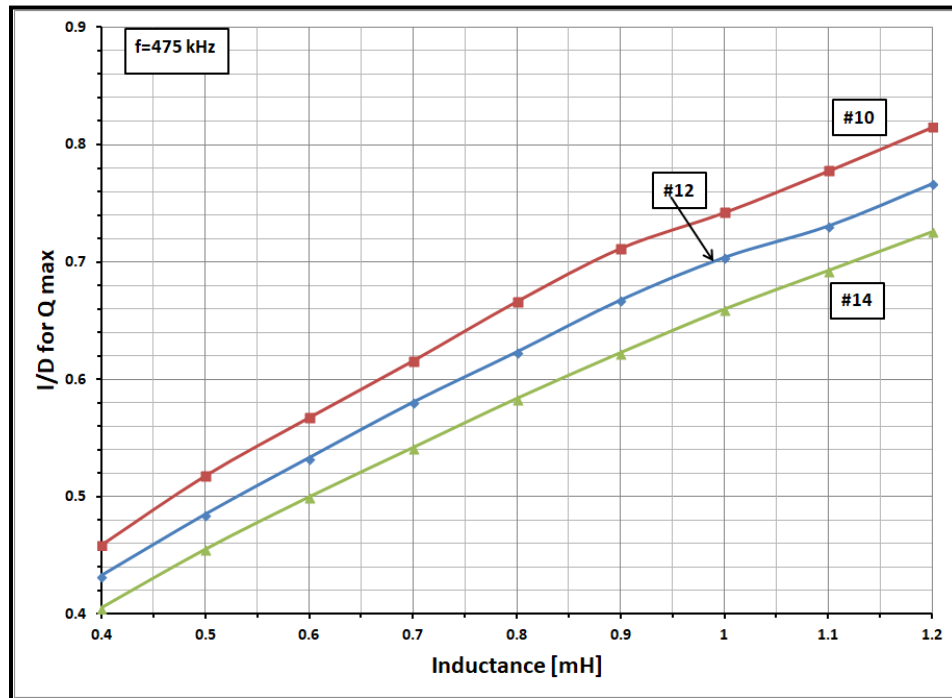


Figure 14A - I/D ratio for maximum Q at 475 kHz.

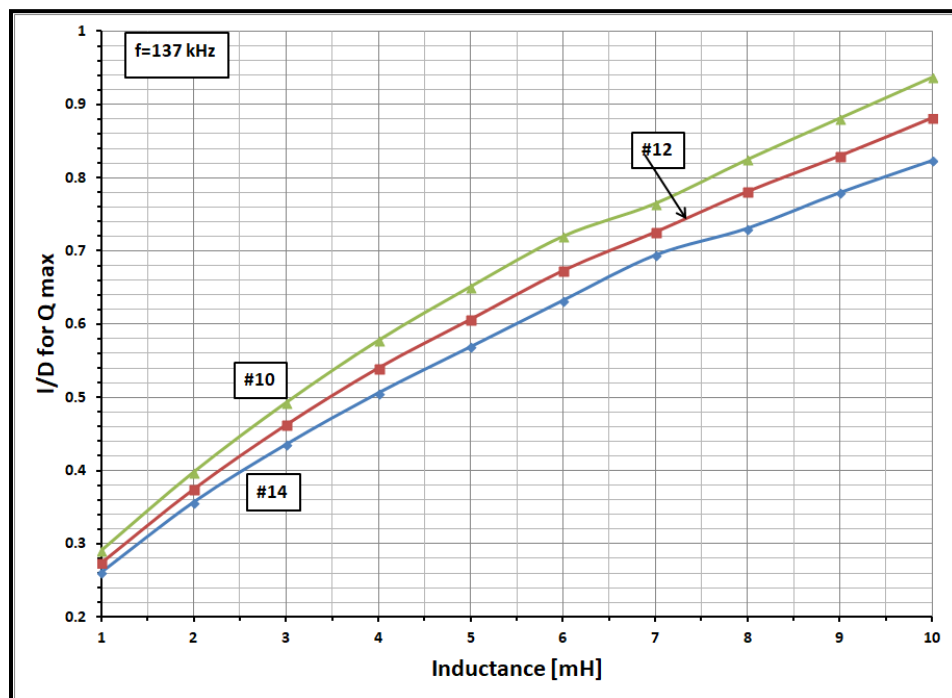


Figure 14B - I/D ratio for maximum Q at 137 kHz.

Figure 15 shows the relationship between f/SRF and I/D at 475 kHz. The 137 kHz graph is very similar to the 475 kHz graph. The lower left end of the graph starts at 400 μH and the upper right end corresponds to 1.2 mH. As inductance increases SRF decreases forcing I/D higher to compensate.

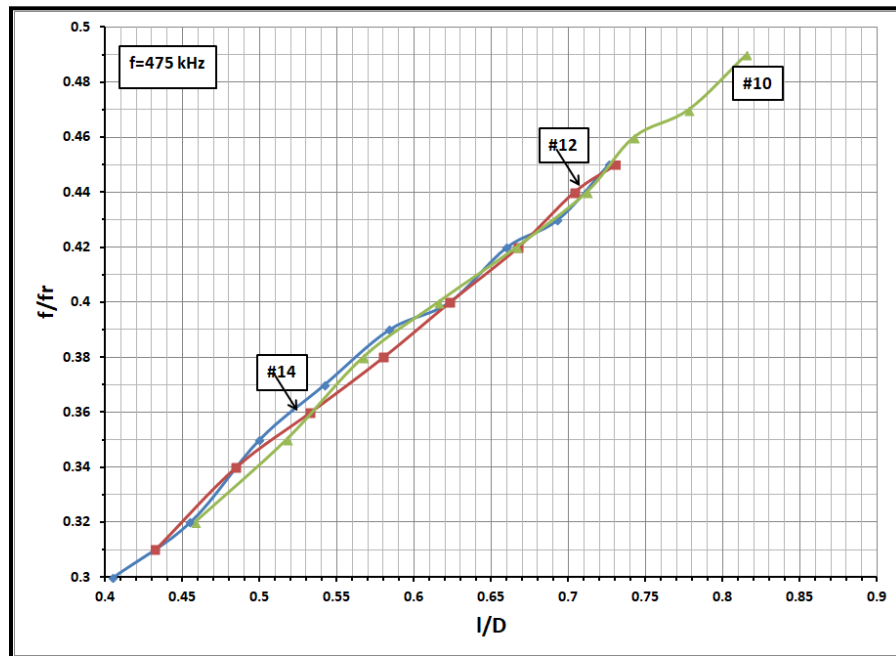


Figure 15- f/SRF versus I/D for 475 kHz maximum Q inductors.

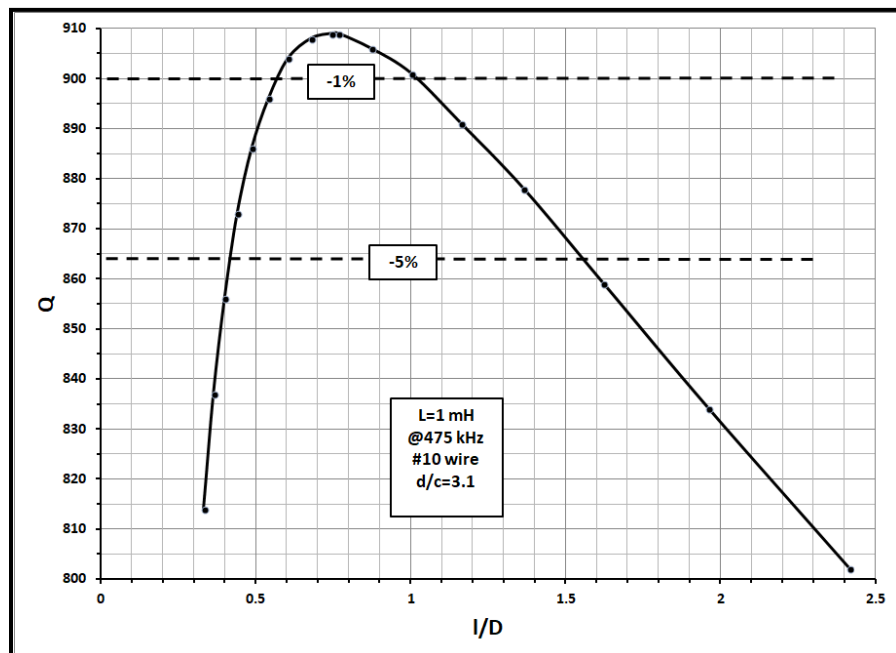


Figure 16 - Q versus I/D example.

We can get a feeling for the sensitivity of Q to I/D from the graph in figure 16. The example is for a 1 mH @475 kHz inductor wound with #10 wire. d/c is constant at 3.1. In agreement with Terman, I/D "is not at all critical".

Conclusions

For small inductors when $f \ll \text{SRF}$, $I/D=0.5$ is good choice. However, the optimum value for I/D is usually greater than 0.45 when an inductor is large enough that self-resonance comes into play which will usually be the case for LFMF antenna tuning coils.

References

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- [2] Frederick Terman, Radio Engineers Handbook, McGraw-Hill, 1943
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