Rudy Severns, N6LF
P.O. Box 589, Cottage Grove, OR 97424-0025 | **rsevens@gmail.com**

Some Thoughts on Radiation Resistance

An explanation of the relationship between radiation and ground loss resistance.

Vertical antennas with radial ground systems have been used since the earliest days of radio. Given 125 years of experience it would appear very unlikely there is anything new to add to the story, but our technical resources have improved dramatically. It is now possible to explore antenna behavior in a detail previously impractical. Computing power has drastically improved along with the creation of excellent free CAD software **[1] [2] [3]** for antenna modeling. Inexpensive vector network analyzers (VNA) are available, making accurate feedpoint impedance (Zi = Ri + jXi) and transmission (S21) measurements in the field much easier.

In this article I want to use some of these capabilities to address a small mystery which has been with us for some 90 years. I will also show how the traditional equivalent circuit model for feedpoint impedance shapes our views on how antennas work and generates some controversy. The discussion begins with the traditional equivalent circuit for the input resistance. Then there is a careful discussion of what is meant by the term "radiation resistance." Finally the "mystery" is shown and the conclusions resulting from it.

Feedpoint Equivalent Circuit

Figure 1 is an equivalent circuit representing the feedpoint impedance, Zi = Ri + jXi, of a resonant (Xi = 0) vertical close enough to ground for the power lost in the soil to be a concern. (This equivalent circuit is typical of antenna texts, such as Kraus, Johnson, Laport, etc. — *Ed.*)

Several quantities are defined. For example, Io is the RMS input current and Ri = Rr + Rg + R_L is the input resistance. The input power is simply: Pi = Io^2 / Ri. Pi is dissipated in two ways: some is radiated (P_r) and the rest is dissipated as heat in the soil (P_g) and antenna conductors (P_L). The radiation efficiency (η) is the ratio of the radiated power to the input power:

$\eta = P_r / P_i$ = Rr / Ri

where Rr is the "radiation resistance" (Rr = P_r / Io^2) and Rg is the "ground loss resistance" (Rg = P_g / Io^2). Usually, P_g is much larger than P_L so for the purposes of this discussion P_L will be omitted. Notice that this equivalent circuit separates the radiated power from the ground loss power by assuming that P_g is not part of P_r. This assumption is built into the equivalent circuit. This small point will be important later. This simple intuitive equivalent circuit has a long history of practical use so you wouldn't think there was much to argue about, but that's not quite true.

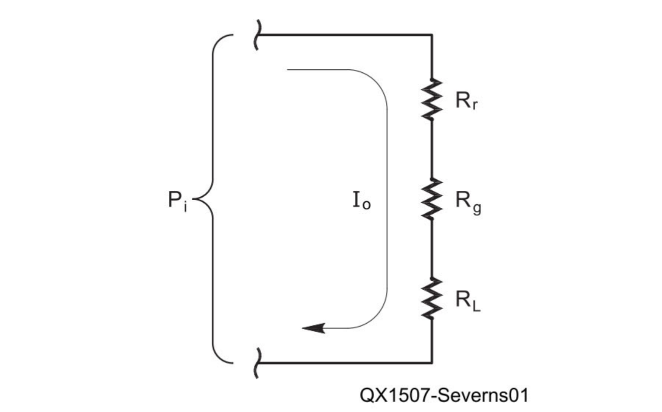

Figure 1 — Feedpoint equivalent circuit when resonant.

One point often misunderstood is the meaning of Rg. Rg is defined as: Rg = P_g / Io^2 but Rg is not a fixed resistance determined solely by Io and the details of the soil and radial system, exclusive of the antenna. The loss in the soil close to the base of a vertical is the result of the electromagnetic fields in the soil which varies with radial distance from the base. This loss varies as the square of the field intensity, so Pg is quite sensitive to small changes: not only the physical ground system configuration and soil characteristics, but also the details of the current distribution on the vertical which can change in subtle ways interactively as the soil or ground system is changed. Rg is just a bookkeeping tool to relate Pg to Io and R_r is another relating Pr to Io. We have to be careful to define what we mean by "radiated power" and what constitutes "ground loss."

Radiation Resistance, Rr

For a lossless antenna in a lossless space the definition of Rr is simple and non-controversial, but for antennas in lossy environments (reality!) things get more complicated. ("Lossless" means that Rg and R_L equal zero — no ground or element resistive losses. See **Reference 5**. — *Ed*.) Looking through the literature for the past 100 years or so, the definitions for Rr in a lossy environment have often been inconsistent or absent. Everybody talks about Rr for a lossless antenna in free space but not so much about the real world.

Rr For a Lossless Antenna

A definition of Rr associated with a lossless antenna in free space, can be found in any antenna book. A typical example is given in the *Radio Engineers' Handbook* by Frederick Terman **[4]**:

"……The radiation resistance referred to a certain point in an antenna system is the resistance which, inserted at that point with the assumed current Io flowing, would dissipate the same energy as is actually radiated from the antenna system. Thus:

$$Radiation\,resistance = \frac{radiated\ power}{I_o^2}$$

Although this radiation resistance is a purely fictitious quantity, the antenna acts as though such a resistance were present, because the loss of energy by radiation is equivalent to a like amount of energy dissipated in a resistance……."

For a lossless antenna in a loss free space the equivalent circuit for the resistive component of the feedpoint impedance is simply a resistor as shown in **Figure 2**.

Rr and Lossy Antennas

Discussions of Rr where the effect of a lossy dielectric in the near-field (soil for example) is considered are not very common and sometimes there are differences between authors. The effect of ground (which is a complex lossy dielectric) on Rr is greatest for antennas close to ground.

The equivalent circuit for the resistive part of the feedpoint impedance in a lossless antenna is simply a resistor but for real antennas, as was shown in **Figure 1**, the model is more complicated. This model has been in use since the early days of radio and is considered "traditional." An explanation relating to **Figure 1** can be found in Edmund Laport, *Radio Antenna Engineering* **[5]**:

"….The antenna resistance therefore is composed of several components which account for the various power losses. The energy lost from the antenna circuit because of the radiation of waves into space is of course the *useful* loss, and that component of antenna resistance which is associated with the radiation of energy is called the "radiation resistance." The efficiency of the antenna system is the ratio of the radiation resistance to its total resistance. In antenna engineering one of the objectives is to make this ratio as large as possible……"

Note that "useful" in "useful loss" is in italics to highlight it. Laport is emphasizing that Rr is derived from the radiated power observed at a distance from the antenna; the local power loss in the soil (P_g) is not included in Pr. As an antenna engineer, his job was delivering a signal to a remote point so that definition is not surprising. Laport is not the only one to make this careful distinction. Johnson, in his *Antenna Engineering Handbook* **[6]**, makes the same point with the following definition for Rr:

"…. R_r = antenna radiation resistance. The antenna radiation resistance R_r accounts for the radiation of useful power….."

The explicit definition of Rr in terms of "useful power" in professional literature suggests that the definition for Rr in a lossy environment may have been a matter of dispute or at least discussion for some time.

Laport describes how one might determine Rr:

"…. The classical original method of computing the radiation resistance of an antenna was to compute its radiation pattern at great distance in terms of field strength and square the field strength at all points on an enclosing hemisphere (in the case of an antenna located near ground). The radiation pattern is then in terms of power flowing outward through the hemisphere, and the integration of power flow over the surface of the enclosing hemisphere gives the total radiated power from the antenna. …….."

This idea is illustrated in **Figure 3**.

This is the procedure I use for computing Rr using values for E and H fields from CAD modeling. At this point we

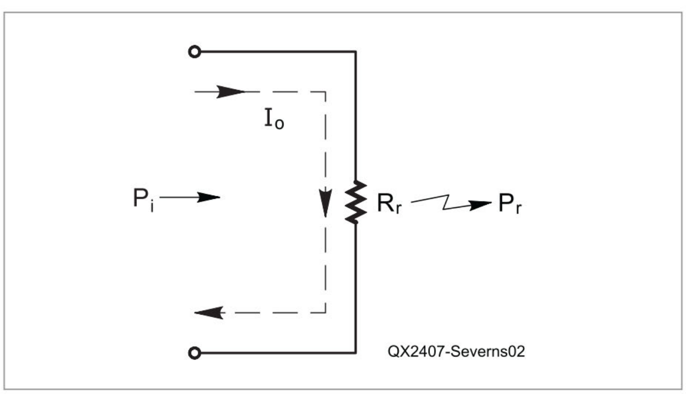

Figure 2 — Feedpoint equivalent circuit for a lossless antenna, P_r=P_i.

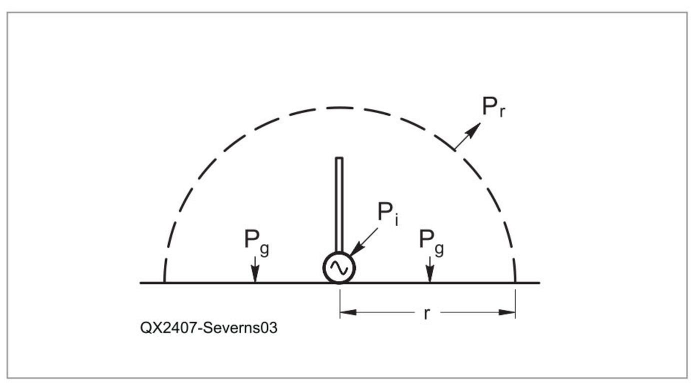

Figure 3 — Derivation of P_r from integration over a virtual surface with radius r.

need to be a little careful. Johnson, Laport and many others define Rr in terms of the far-field, distant from the antenna. P_g then represents all the power lost from the antenna base out to infinity. This is not quite the same as how I have defined Pg in previous articles **[7]**. When modeling verticals with radial ground systems one common exercise is to vary the number and length of the radials. In a general way more and longer radials reduce the value of P_g. In fact when using numerous radials a point of vanishing returns is reached where P_g decreases very little with an increase in radial length. Typically this occurs for radial lengths greater than ≈ 0.4λo. (λo is the free space wavelength at the operating frequency.)

When performing the power integration over a hemisphere, I typically use the field components at a radius (r) equal to 0.5λo which is the radius within which the ground system significantly affects the radiated power. Restricting r to be less than infinity greatly complicates the calculation because of the need to use the phase and amplitude of both the E and H fields at radius r for the calculation. For the infinite r case, modeling software usually provides the "average gain" which is P_r integrated at infinity divided by P_i. The difference between the two calculations is usually quite small but allows for the case where radiation closer than infinity might be considered "useful."

Alternate Definitions for Radiation Resistance (Rr')

Terman is very explicit:

$$\text{“}\, Radiation\, resistance = \frac{radiated\ power}{I_o^2} \,\text{”}$$

Radiation resistance is defined by radiated power and Io. There seems to be no wiggle room here. It could be argued that any "radiated power," whether into space or into the ground, should be counted towards P_r and Rr derived from that value. It would appear from Figure 3 that Pg represents power "radiated" from the antenna into the ground and should be counted as part of P_r. This appears to be a reasonable intuitive observation. Carrying this idea forward we might say that $P_r' = P_r + P_g$ and therefore Rr' = Rr + Rg. The key assumption is that the power dissipated in the soil is "radiated" from the antenna. It turns out that is substantially not the case. For antennas close to ground only a small part of the power dissipated in the soil (P_g) is "radiated" from the antenna!

Some Explanation

We need to step back and look closely at the EM fields close to the antenna. These fields have energy-storing reactive near-field (or induction field) and radiating field components. Mathematical expressions for the E-field have three amplitude terms, one varying as 1 / r, a second as 1 / r^2 and a third as 1 / r^3, where r is the distance from the source. Kraus **[8]** has a very helpful graph showing the relative amplitudes of these terms as a function of distance r / λ (radius in wavelengths) from a small antenna shown in **Figure 4**. The

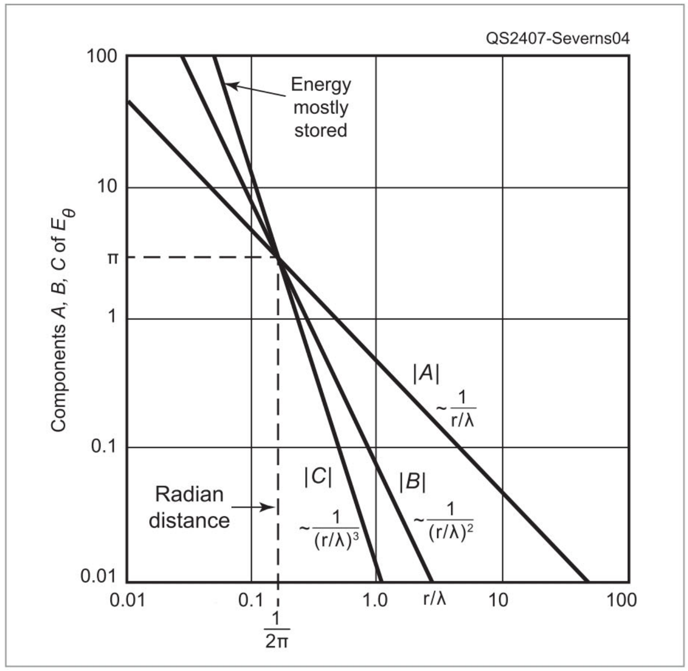

Figure 4 — From Kraus, *Antennas*, 2nd edition, Figure 5-6 **[8]**.

radiator in Kraus is assumed to be a very short electric dipole with a constant current distribution. An actual antenna with some physical height will be an end-to-end stack of these dipoles with an overall current profile suitable for the antenna. This means that at any point away from the antenna the fields will be a linear superposition of the short-dipole fields. This complicates the fields in the immediate region of the antenna but we can still use the simpler expressions to understand the general nature of the fields.

The key point to note is that the 1 / (r / λ^2) and 1 / (r / λ^3) terms are *non-radiating*, they represent energy stored in the near-field. Only the 1 / (r / λ) term represents radiation. This graph is for E_θ at any angle. For this discussion I'm assuming θ=90° and the ground loss is proportional to E_θ along the ground surface.

One more point to note, Figure 4 is in terms of the relative E-field amplitude but any loss or radiated power will vary as the square of the amplitude which expands the Y-axis scale, i.e. 0.1 → 0.01 or 10 → 100, etc. This greatly increases relative differences. For r / λ < 1/2π (≈0.16) the loss is dominated by the 1 / r^2 and 1 / r^3 terms and little ground loss is due to the radiation term 1 / r. From the r / λ = 1/2π point to my usual boundary at R / λ = 0.5, the 1 / r term dominates but by that point the field amplitude is down by orders of magnitude, so the loss is still relatively small.

This sounds reasonable, but we need to be more careful. When a radial ground system is used, the field intensities in the soil close to the base are greatly attenuated, which is why we use a radial ground system. However, as one goes away from the base the space between the radial wires increases and the field attenuation in the soil is reduced. One could argue that further from the base, where the radiation term [1 / (r / λ)] becomes more significant that the proportion of

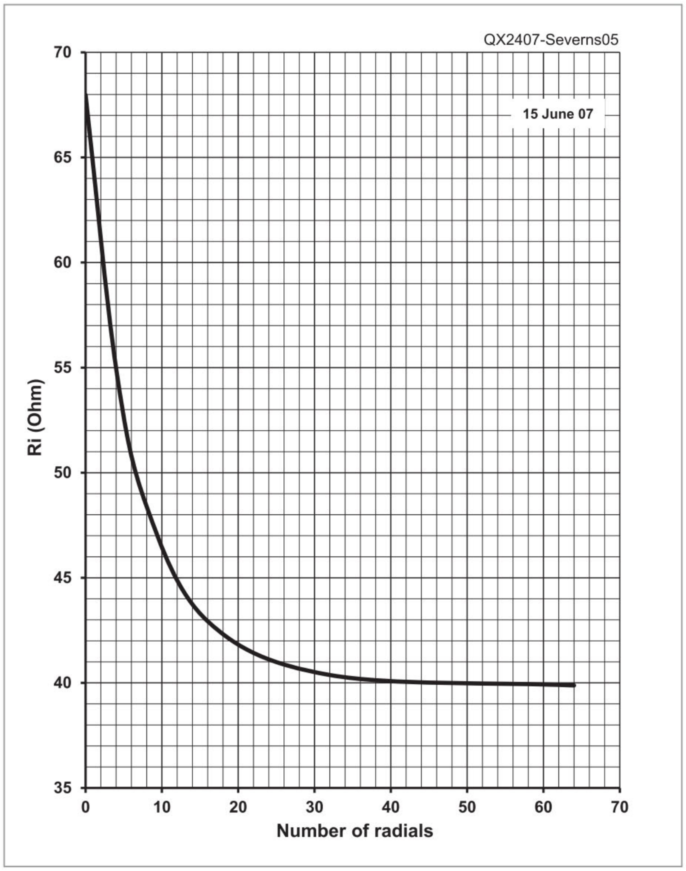

Figure 5 — Experimental values for Ri at the base of a 1/4-wave vertical at 7.2MHz.

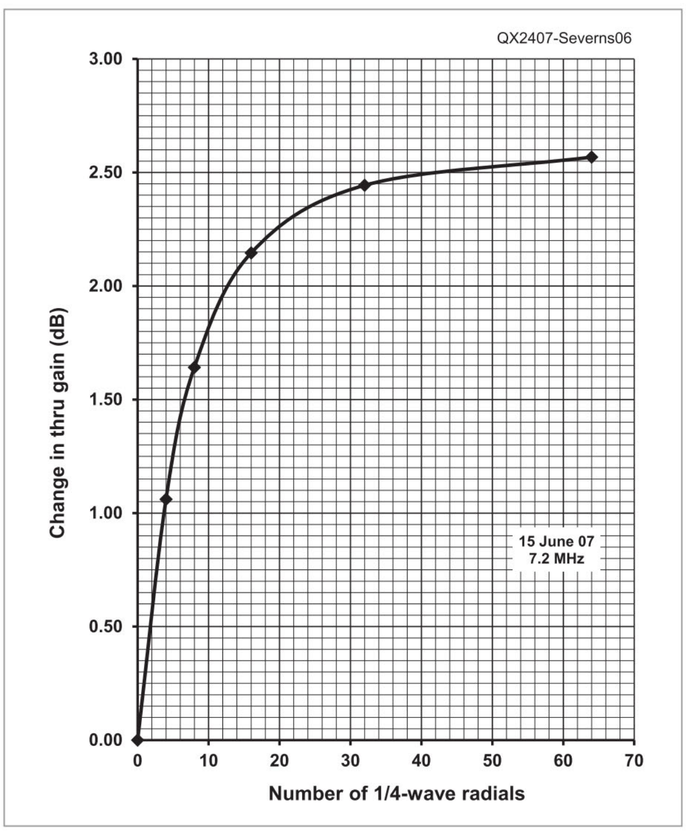

Figure 6 — Change in far-field signal strength as a function of the number of 1/4-wave radials.

radiated power in P_g will increase.

There is another complication. Soil is a lossy dielectric. When the soil is close to the antenna, Ri will increase reflecting the losses but there is also a non-lossy effect on Ri. As I showed in my 2015 *QEX* articles, **[7]** even if the nearby dielectric is lossless, just its presence in the near-field of the antenna changes Rr. It could be argued that the volume of soil within the near-field is part of the antenna.

There are alternate views on the definition of Rr with a lossy dielectric in the near-field. The traditional one of Pr = useful power has intuitive appeal and practical use. The alternate views are an interesting intellectual exercise, but I don't see them as having a lot of practical use, at least for antennas in a lossy environment, although that is certainly open to debate.

A Small Mystery

Does all this quibbling over the definition for Rr matter or is it an insignificant detail? What follows suggests it does matter. P_i and Ri are easily measured at the feedpoint, but Rr and Rg cannot be separately determined from a feedpoint measurement. Determining Rr and Rg requires knowledge of the E and H fields near the antenna. In the past this required some daunting math and/or difficult measurements so the problem was simplified by assuming Rr equals the Ri value of a lossless vertical over a perfectly conducting plane. For a lossless resonant $\lambda/4$ vertical over perfect ground Rr is very close to 36 Ω. In the traditional view, if we know Ri

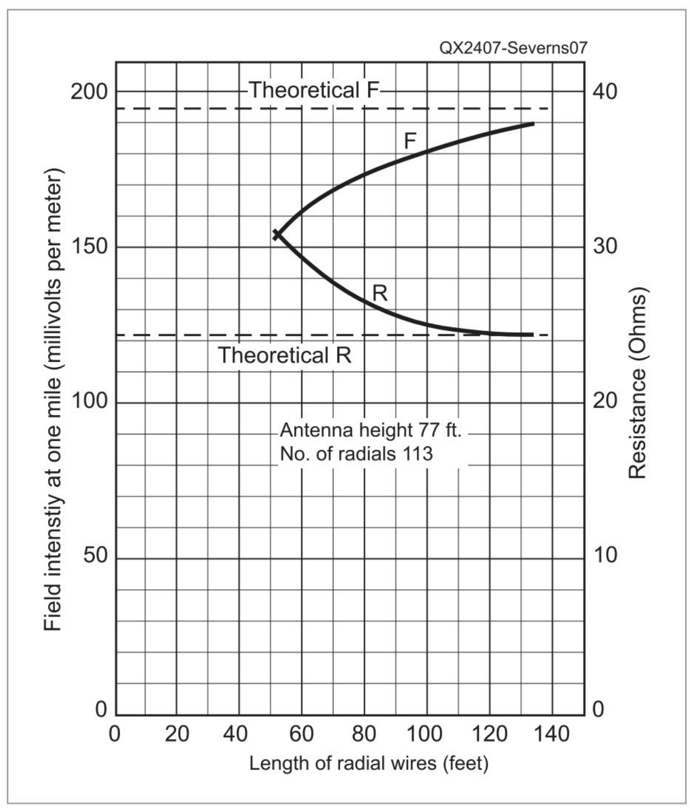

Figure 7 — Figure 38 from Brown, Lewis and Epstein [**10**].

and R_L is small then Rg ≈ Ri - 36 Ω. As the ground system is improved, (e.g. more and/or longer radials and/or better soil) Rg will become smaller and the measured value of Ri should approach 36 Ω. **Figure 5** shows a typical measurement **[9]**.

In this example the radial length was ≈λ/4. What we appear to be seeing is Rg decreasing as more radials are added but there is a limit. By the time there are ≈40 radials, Ri has flattened out at ≈40 Ω implying Rg ≈ 4 Ω and η ≈ 36 / 40 = 0.90. The Ri curve seems to show that using more than 40 radials in this case will result in no increase in radiated power. However, during that particular experiment a transmission measurement was made (the change in signal received at a distant point) which is shown in **Figure 6.**

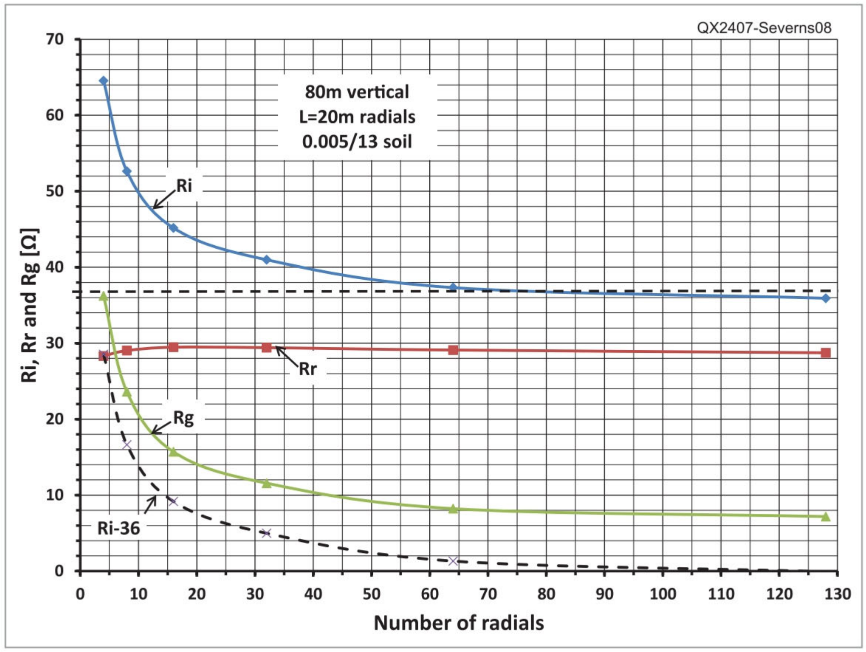

Figure 8 — Ri, Rr and Rg versus number of radials.

Figure 6 shows continuing improvement past the point where Ri has leveled out in **Figure 5**. The two figures do not agree. This is *not* a new observation for such measurements.

Figure 7 is taken from the 1937 IRE paper by Brown, Lewis, and Epstein (BLE) **[10]** which is a reference for many subsequent broadcast and amateur antenna ground system designs. In **Figure 7**, R corresponds to Ri and F is signal strength (mV/m) at a distant point. In this example the number of radials was fixed at 113 but the length varied from 50 to 135 feet. The test frequency was 3 MHz where 50 – 135 feet corresponds to radial lengths of 0.15λ - 0.41λo. The height of the test antenna was 77° rather than 90° so the theoretical Rr was lower. Like Figures 5 and 6, R flattens out with improvement in the ground system, but F continues to increase. What is happening? I am not the first person to look at the BLE graphs and raise questions. Dave Gordon-Smith, G3UUR, in his 2016 *RadCom* article **[11]** raised several questions. In particular he questioned the conventional belief that Rr for a λ/4 vertical is 36 Ω independent of the soil and ground system. *EZNEC* modeling can be used to explore that question.

Derivation of P_r and P_g from EM Field Data

It is possible to derive Pr and Pg (and from them Rr and Rg) from the EM field near the antenna as illustrated in **Figure 3**. When the E and H field components on a given virtual surface are known the outward flowing power density (S in W/m^2) can be determined from the vector product of the E and H field components. By summing S over the surface, the total power passing through that surface for a given input power can be determined. For Pr calculations, r was set to be approximately the outer edge of the reactive near-field which is ≈λ/2 for a λ/4 vertical. The near-field decays exponentially so there is no "hard" edge but r = λ/2 is a reasonable compromise. P_i and Ri are provided directly by the software. Of course, we still have to determine the fields on the surfaces. In the past that was often a serious mathematical exercise and/or an involved experimental procedure but we are more fortunate, CAD modeling can generate this information with minimal effort. For an arbitrary antenna this can still be a challenge but things are much simpler with a ground-mounted vertical because the fields are azimuthally symmetric. We only need to calculate

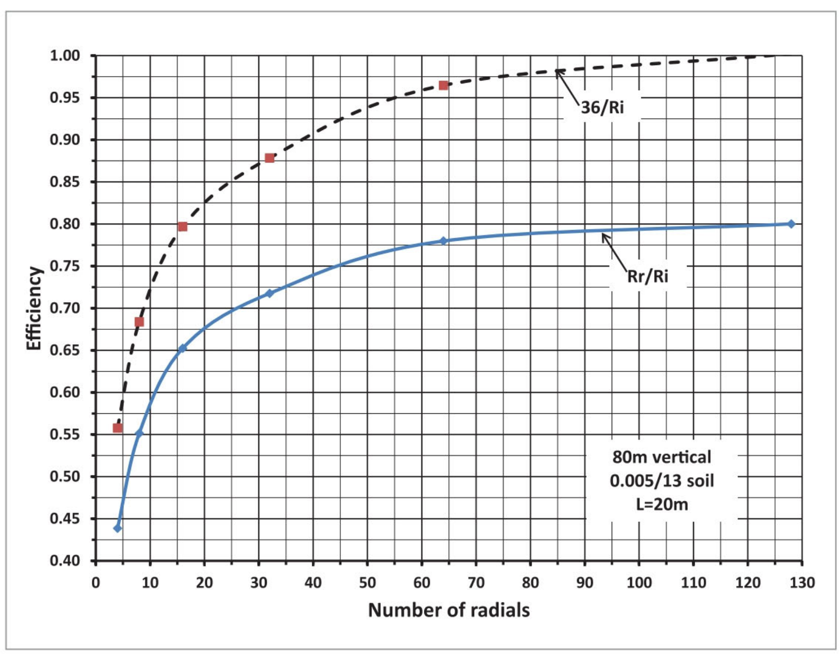

Figure 9 — Efficiency versus number of radials.

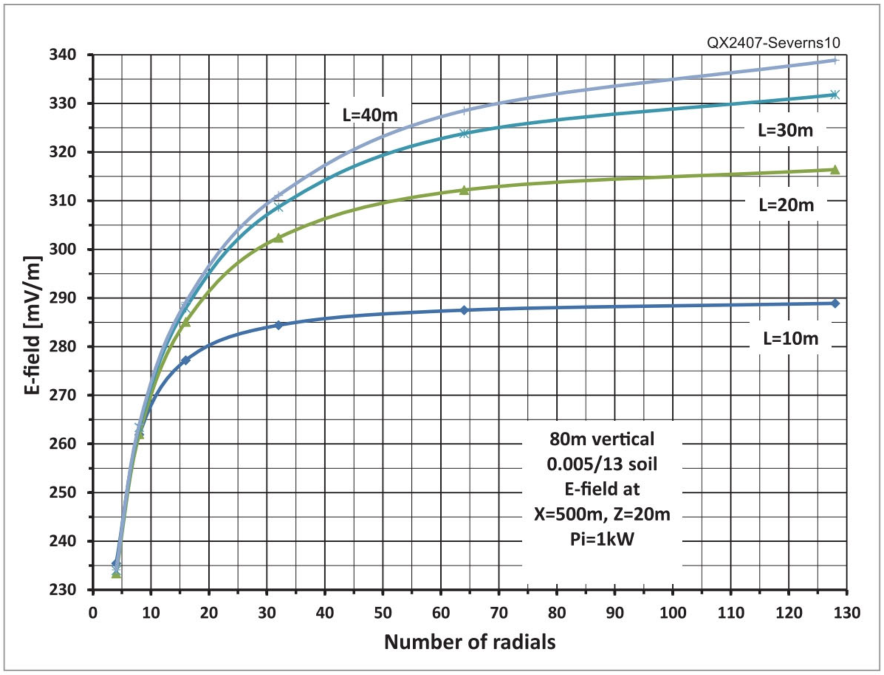

Figure 10 — Signal strength [mV/m] versus number of radials for several radial lengths.

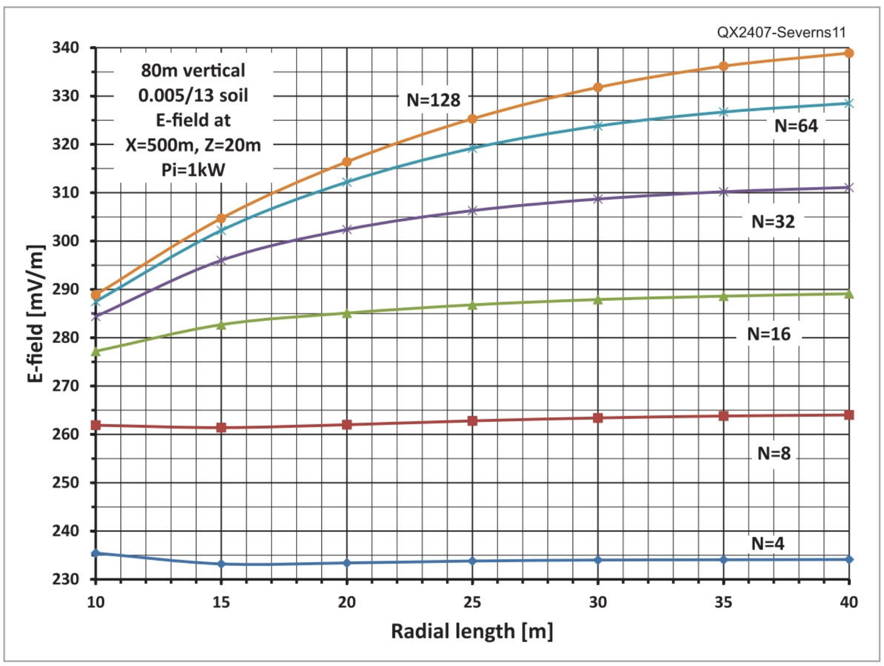

Figure 11 — Signal strength [mV/m] versus radial length for various radial numbers.

the values at a number of points along a 90° arc in the X–Z plane. These values can then be entered into a spreadsheet to calculate close approximations for P_r and P_g for a given input power and from them Rr and Rg may be derived.

A Modeling Example

The modeling frequency was 3.6MHz. The vertical and the radials were #12 wire. Radial lengths (L) were varied from 10 m to 40 m (≈0.12λ to 0.48λ) over four different soils: free space (σ = 0, εr = 1), poor (σ = 0.002, εr = 13), average (σ = 0.005, εr = 13) and very good (σ = 0.030, εr = 20) where σ is the conductivity in S/m and εr is the relative dielectric constant. The radials were buried –1" and the feedpoint located at the base of the vertical at the junction with the ground system. The length of the vertical was adjusted to maintain resonance as soil and radials were altered. *EZNEC* Pro/4+ v.7.0 **[1]** with the *NEC5* engine combined with *AutoEZ* **[3]** were used for the modeling.

Figure 8 shows contours for Ri, Rr, Rg, and Ri – 36 Ω as a function of radial number using 20 m (≈λo/4) radials over average ground. The Ri contour is pretty much what we would expect, starting at a high value with four radials and steadily decreasing as the radial number increases, flattening out and approaching 36 Ω with 128 radials. If Rr = 36 Ω then the Rg contour would look like the dashed line (Ri – 36) which indicates that Rg≈0 for 128 radials. But when Rr is derived from the radiated power, we get Rr ≈ 30 Ω, substantially less than 36 Ω. The Rg contour shows Rg ≈ 7 Ω when 128 radials are used. Note that Rr peaks at N ≈ 20 and actually goes down a small amount for more numerous radials.

Using these values for Ri and Rr we can graph the efficiency as shown in **Figure 9** with contours for Rr / Ri and 36 / Ri. If Rr were actually 36 Ω then the efficiency with 128 radials would be close to 1 (100%) but in reality the efficiency is significantly lower, ≈0.80.

"Radiation efficiency" is not a common metric. It is more usual to graph the field strength at some distant fixed point (X, Z) as shown in **Figures 10** and **11**. The effect of longer radials on signal strength at a given distance (X = 500 m and Z = 20 m and Pi = 1kW) is illustrated in Figure 10 for different numbers of radials. Signal strength can also be graphed versus radial length as shown in **Figure 11**.

Figures 8 and **9** assumed 20 m radials. **Figure 12** shows the effect of longer radials on Rr. When 128 40 m radials are used, Rr almost reaches 36 Ω.

Figure 13 shows Rr when 20 m radials are used over different soils ranging from free space to very good. Rr is not a very strong function of radial number but rises steadily with soil conductivity approaching 36 Ω for very good soils.

Despite the unexpected dance between Rr and Rg, the Ri and signal strength graphs derived from modeling correspond well to field measurements. This discussion has assumed a ¼-wave resonant vertical. However, verticals with

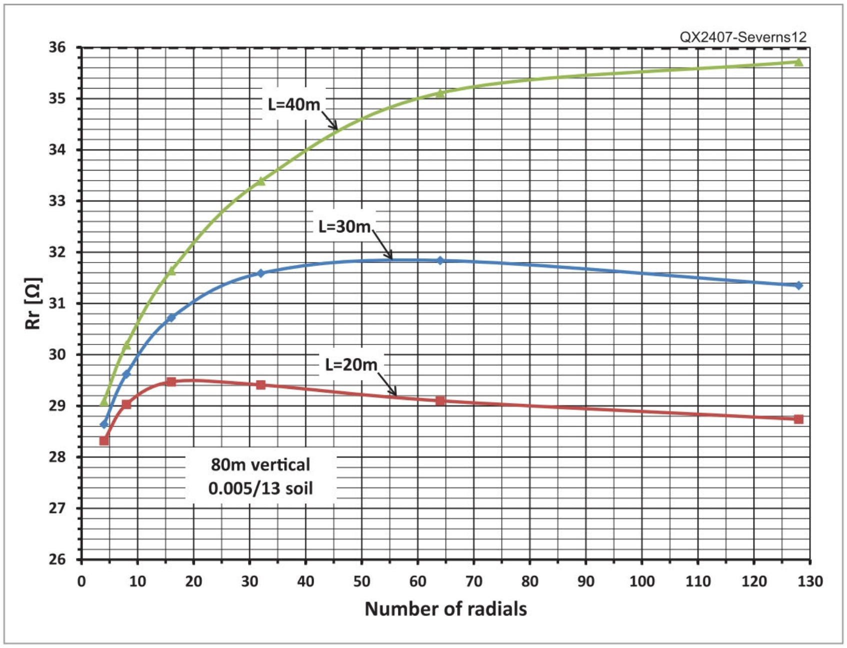

Figure 12 — Rr versus number of radials for different radial lengths.

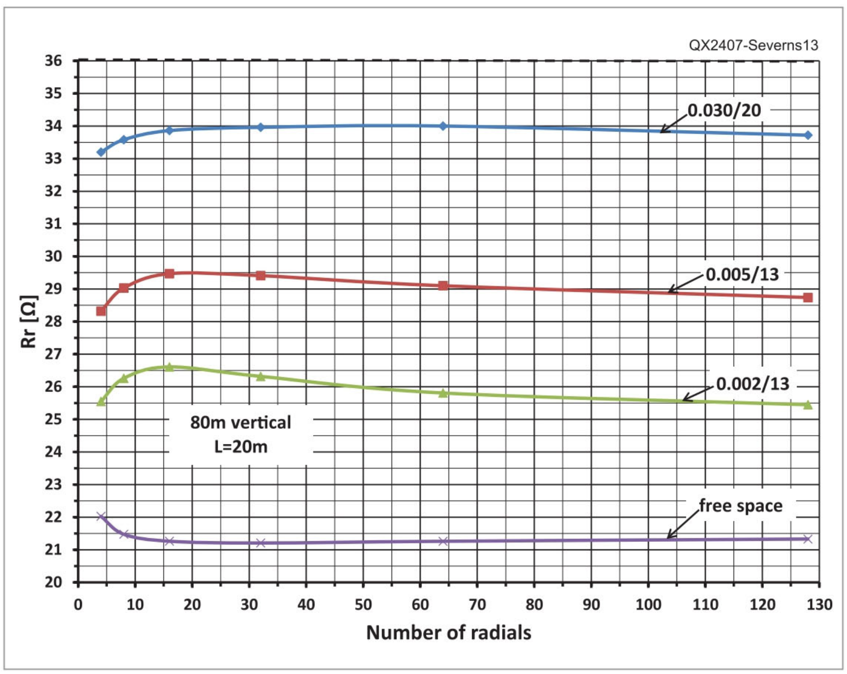

Figure 13 — Rr versus number of 20 m radials for different soils.

other heights also show the same behavior, just with different values. A more detailed discussion can be found in my 2015 article **[7]**.

Ri Less Than 36 Ω

Occasionally measurements of feedpoint impedance for a ¼-wave vertical will yield values for Ri < 36 Ω. Such measurements tend be dismissed as instrument errors but modeling and analysis both show that Ri < 36 Ω is possible under some conditions. It is also possible for Ri to increase with radials longer than λ/4. Again this does not fit the traditional model where increasing Ri would be associated with increasing Rg and reduced radiation efficiency except that the field strength is still increasing as shown in **Figure 12**.

Figures 14 and **15** show Ri as a function of radial length for 32 and 128 radials over three different grounds ranging from poor to very good. Note that Ri decreases and the levels out with increasing length up to roughly L = 20m, ≈λ/4. For longer radials, Ri increases with length which certainly does not fit the traditional Rr = 36 Ω view. Amateurs generally use radials ≤λ/4 so this behavior would not usually be seen. It has however, been predicted by James Wait **[12]** in a mathematically dense technical paper. The G3UUR article **[11]** also supports that conclusion.

In Figure 12 Ri > 36 Ω but, as shown in Figure 15, when many more radials are used, Ri can be less than 36 Ω. This effect seems to show up when large numbers of long radials are used over average or poor soils.

Conclusions

Modeling predicts that longer and/or more numerous radials and/or better soil generally provide better efficiency with the exception of very sparse radials fans (4-8 radials), which is completely in line with experience. The predicted behavior of Ri with radial length and number also agrees with experience. *The difference is the relationship between Ri, Rr, and Rg.*

Modeling makes the case that the traditional view (Ri = Rg + 36 Ω) for a resonant ¼-wave vertical is not correct. Rr is usually lower than 36 Ω, often much lower, depending on the number and length of the radials, soil electrical characteristics and/or frequency. Estimates for Rg derived from the traditional relationship will be too low and actual radiation efficiency lower than anticipated. This discussion also applies to verticals with height less than λ/4. In general, Rr will be less than that for the vertical over perfect ground with the value for Rr converging to the perfect ground value when a large dense radial field is used.

Acknowledgements

I'd like to express my appreciation to Greg Ordy, W8WWV; Dan Maguire, AC6LA, and Al Christman, K3LC, for their help. Greg contacted me regarding Ri measurements he had made on a vertical array at K3LR stimulating me to look at this matter again. Dan was very helpful suggesting ways to use *AutoEZ* to significantly reduce the computational effort

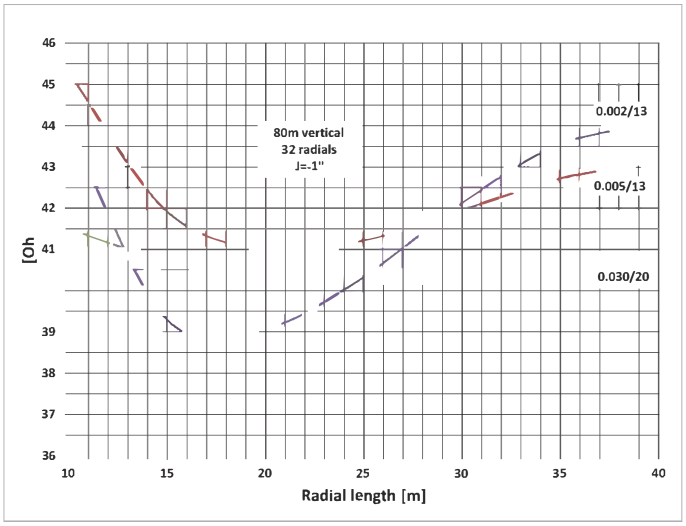

Figure 14 — Ri versus radial length L with 32 radials.

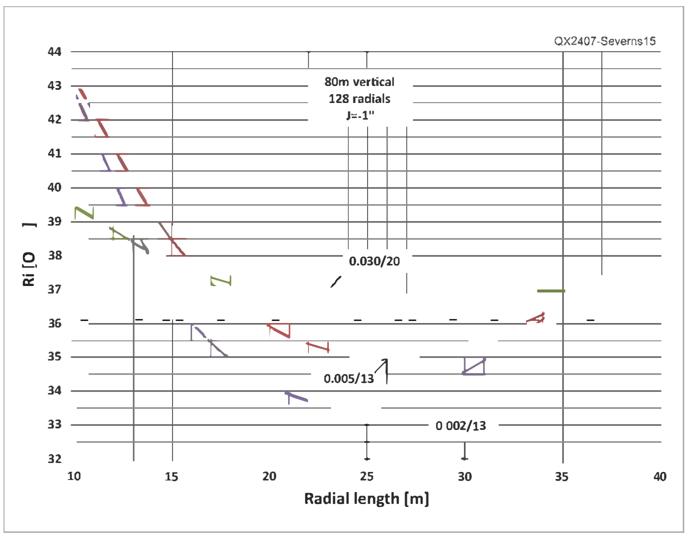

Figure 15 — Ri versus radial length L for 128 radials.

and commenting on the article as it progressed. Al was a very patient and encouraging reviewer.

References

[1] *EZNEC*, **eznec.com**

[2] *4nec2,* **qsl.net/4nec2**

[3] *AutoEZ,* **ac6la.com**

[4] Frederick Terman, *Radio Engineers Handbook*, McGraw-Hill, 1943, page 784

[5] Edmund Laport, Radio Antenna Engineering, McGraw-Hill, 1952, pages 2 and 8

[6] Richard Johnson, *Antenna Engineering Handbook*, 3rd edition, McGraw-Hill, 1993, page 24-5

[7] R. Severns, N6LF, Radiation and Ground Loss Resistances in LF, MF and HF Verticals, *QEX,* Jul/Aug and Sept/Oct 2015

[8] John Kraus, W8JK, *Antennas*, 2nd edition, McGraw-Hill, 1988, page 210

[9] R. Severns, N6LF, Experimental Determination of Ground System Performance for HF Verticals, *QEX*, seven parts Jan 2009-Jan 2010

[10] Brown, Lewis and Epstein, Ground Systems as a Factor in Antenna Efficiency, *IRE Proceedings*, June 1937, figure 38, page 779

[11] Dave Gordon-Smith, G3UUR, Ground Systems: a fresh look at an old classic, *RadCom,* December 2016, pp. 74-75, 80

[12] James Wait, Characteristics of Antennas Over Lossy Earth, Collin and Zucker, *Inter-University Electronic Series Vol. 7*, 1969, pp. 386-437